

Background of the Author

The author, Eric Holcomb, has an MS degree in aerospace engineering, and worked as an engineer for The Boeing Company for nearly 20 years. Eric now pursues his own business activities, enjoys playing chess, and serves as business manager for Northwest Chess magazine.

An Inconvenient Chessic Coincidence?

By Eric Holcomb
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In the author's previous article, "So How Many Chess Board Positions Are There?" the author noted that there is a remarkable coincidence involving one estimate of the number of chess board positions.

The nature of the coincidence can now be revealed, and it turns out it's a remarkable coincidence between chess and global warming! In particular, the number of carbon dioxide molecules in the Earth's atmosphere is nearly equal to a reasonable upper limit on the number of chess board positions that can be constructed without resorting to pawn promotion to gain extra pieces! The "magic number" is about 4.1529×10^{40} , where 10^{40} is the number "1" followed by 40 zeroes – a very large number indeed!

See the previous article in the August magazine or on the Northwest Chess website for a complete explanation of this number of chess board positions and the assumptions involved in calculating the number.

Is this just an amazing coincidence? Did "Mr. Inconvenient" Al Gore invent the game of chess in an earlier life? (He did invent the Internet, you know!) Or, perhaps the chess goddess Caissa is trying to tell us that this is a limit which should not be exceeded!! (See the "Chess Carbon Limit" graphic.)

Estimating the number of carbon dioxide (CO₂) molecules in the Earth's atmosphere follows the basic principles of gas chemistry and physics as taught in high school and college. Details will be posted on the Northwest Chess website, with a brief summary in this article.

The "effective volume" of the Earth's atmosphere is about $4.192 \times 10^{18} \text{ m}^3$ (cubic meters), calculated as the surface area of the Earth ($5.094 \times 10^{14} \text{ m}^2$) times the "effective thickness" of the atmosphere (8,228 meters, or about 27,000 ft). The effective thickness and volume are what the thickness and volume of the atmosphere would be if the entire atmosphere were at sea level conditions. These can be estimated from "standard atmosphere" models. The effective thickness includes a small correction for the volume of land above sea level, and is consistent with the total dry mass of the Earth's atmosphere ($5.1352 \times 10^{18} \text{ kg}$) as given in the Wikipedia article on this subject.

Readers may recall that the density of a gas is proportional to pressure and inversely proportional to temperature, and that there's also a "gas constant" in the equation, which is the same for all gases if expressed in terms of "moles." When the calculation is done using the correct units, the

average density of the Earth's atmosphere at sea level is 42.293 gram-moles/m³. (Multiplying by the average molecular weight would give the density in grams/m³.)

The total number of gram-moles of gas in the atmosphere is 1.773×10^{20} (the average density times the effective volume). The carbon dioxide concentration is increasing due to the combustion of fossil fuels, and is currently estimated at 384 parts per million (ppm) by volume, which is also the molar concentration. The number of gram-moles of CO₂ in the atmosphere is about 6.807×10^{16} ($0.000384 \times 1.773 \times 10^{20}$). Finally, one gram-mole contains "Avagadro's Number" of molecules, or 6.023×10^{23} molecules per gram-mole. (Remember that one from chemistry class?) Thus the number of molecules of CO₂ in the atmosphere is estimated as 4.100×10^{40} (the number of gram-moles of CO₂ times Avagadro's Number).

So there you have it ... At the current (2008) carbon dioxide concentration of 384 ppm, the remarkable coincidence between chess board positions and carbon dioxide molecules becomes apparent!! The concentration of CO₂ is increasing at a rate of 1.8 ppm per year ... The two numbers will be the same in only about 3 years when the concentration reaches 389 ppm.

In terms of mass, the current atmospheric level of CO₂ is equivalent to 818 gigatons of carbon, the unit of measure normally used by climate scientists. The September 2006 *Scientific American* article "A Plan to Keep Carbon in Check" quotes the then-current amount of atmospheric carbon as 800 gigatons, just 2% less than the present estimate. Even if this lower figure was correct for the year 2006, at the present rate of increase, the "magic number" will still be reached in about 6 years, by the year 2014!

Chess Move Sequences

The author also promised a discussion of the number of possible chess move sequences (chess games), often estimated as 10^{120} , greater than the number of atoms in the known universe!

The number 10^{120} is known as the "Shannon number," and was estimated by assuming a typical 40-move chess game, and a game tree "branching factor" of 1,000 (10^3) per full move, or 31.6 (square root of 1,000) per ply, or half-move. Of course many chess games last longer than 40 moves, and many chess game positions have more than 31 or 32 possible legal moves. The 1994 Victor Allis Ph.D. thesis quoted the number 10^{123} , obtained by using a branching factor of 35 instead of 31.6.

The number of possible chess move sequences is infinite if games are not terminated due to repetition of moves, or something like the 50-move draw rule. The Shannon and Allis numbers are very crude estimates that are likely too low because they only account for the first 40 moves of a game. On the other hand, if one assumes that only about three moves in any given position would be considered "sensible" by a Grandmaster, then the number of "sensible" 40-move GM games is only about 10^{40} , not accounting for blunders!

Because the "game tree" branches out so rapidly, an exact computer calculation and analysis of chess moves is only practical for about 11 or 12 ply in most positions, including the starting position. (Deeper and/or faster analysis is possible by "pruning" algorithms, but this gives up the

ability to enumerate all possible move sequences.) For the chess starting position, François Labelle at UC Berkeley reports 2,097,651,003,696,806 possible chess games out to 11 ply, an average branching factor of 24.7. (By the way, there are 362,290,010,907 of those games, or 0.017%, that end in checkmate!)

All of the 10^{120} or more possible chess move sequences must transpose into the much smaller number of possible board positions, which as discussed in the author's previous article, likely does not exceed 10^{50} , even with pawn promotions. Although nearly all chess players are aware of some possibilities for transposition of moves, normally they don't give it much thought, but it should now be obvious that transposition of moves is extremely important! The top computer chess programs use transposition tables to avoid duplicate analysis of move sequences, especially in endgame positions.